On the classification of (2,1) heterotic strings

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Abstract

We classify all untwisted (2,1) heterotic strings. The only solutions are the three already known cases, having massless spectra consisting either of 24 chiral fermions, or of 24 bosons, or of 8 scalars and 8 fermions of each chirality.

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1. Introduction

Closed string theories with N=0 or N=1 world-sheet supersymmetries can be completely classified in their maximal ("critical") dimension, assuming only (super)conformal and modular invariance. This classification yields a single (0,0) string, four (1,1) strings (IIA, IIB, plus two theories without space-time supersymmetry), and nine (0,1) strings (the two heterotic superstrings, plus seven non-supersymmetric theories). In this paper we want to extend these results to (a subclass of) N=2 heterotic strings.

The basic idea behind such classifications is that in the maximal dimension at least one of the chiral sectors of the underlying two-dimensional conformal field theory has its central charge saturated by fields with space-time indices, leaving no room for an unknown internal conformal field theory. The only non-trivial structure one can have in this saturated sector are then the spin-structures of the world-sheet fermions, which have known modular transformations among themselves. In such a situation it is always possible to map the modular invariant partition to a meromorphic one (*i.e.* to a character of a rational conformal field theory with a single primary field; such theories have $c = 8k$). If the central charge of this meromorphic conformal field theory is 8, 16 or 24, one can use the available classifications of meromorphic conformal field theories to determine all possibilities for the unknown chiral internal sector. The (1,1) strings map to $c = 16$ theories, whereas the (0,1) strings map to $c = 24$ theories. In the former case there are no internal sectors, but there are several ways of combining the spin structures, which are easily read off from the two meromorphic $c = 16$ theories, $E_8 \times E_8$ and $Spin(32)/\mathbf{Z}_2$. In the latter case all possibilities for the internal $c = 16$ conformal field theory as well as all ways of combining it with the NSR spin-structures can be read off [1] from the list of possible meromorphic $c = 24$ conformal field theories [2].

Similar considerations should apply to strings with N=2 world-sheet supersymmetry. Indeed, Ooguri and Vafa [3] gave a classification for N=2 strings under certain assumptions. However they made the unnecessarily restrictive assumption that the internal conformal field theories are essentially torus compactifications. Modular invariance then only allows the 24 Niemeier lattices for the right sector of (2,0) strings and the E_8 torus for (2,1) strings. Later [4-7] more general solutions were found in the latter type of theories, but without claims to completeness. The classification of (2,0) strings is easily completed by replacing the 24 Niemeier lattices by any of the 71 meromorphic conformal field theories enumerated in [2] (many of which have been explicitly constructed, see *e.g.* [8,9]).

In [5],[6] and [7] target spaces of N=2 strings were constructed. A large variety of such theories was uncovered. The identification of the exact nature of these target spaces was aided by a conjecture that they provide world sheet theories of critical strings (as well as world volume theories for 3-d membranes). This realized the idea proposed in [10]. It is of interest to arrive at a complete classification of these theories.

There are two additional features in N=2 heterotic strings that can complicate the analysis: the moduli integration in the left (N=2) sector, which identifies the spin structures, and the need for a "null current" reduction of an extra space and time direction

in the right sector. When fully exploited, these features may give rise to additional possibilities of combining left- and rightmovers (Z_n-strings [11]) or twisted boundary conditions in the null directions [5]. Here we will consider only the simplest case, where such additional twists are absent. Perhaps our analysis can be generalized to cover the other cases as well.

Under the assumptions stated above the left sector is modular invariant by itself, and requires no further discussion. The null current reduction requires only slightly more care. A (2,2) string has a (real) critical dimension equal to four, and the no-ghost theorem requires the space-time metric signature to be $(+, +, -, -)$. To build heterotic strings, one chiral sector of such a theory has to be combined with N=0 or N=1 chiral sectors. The presence of two time directions in the N=2 string target space requires the same in the N=0 or N=1 sectors. Furthermore the presence of a $U(1)$ gauge symmetry in the N=2 sector requires the same in the N=0 or N=1 sectors [3] (in the case of N=1, supersymmetry requires in addition to the vector current also a fermionic current in that sector). Ooguri and Vafa showed how these two changes essentially cancel each other: the extra symmetries lead to extra ghosts, whose contribution to the conformal anomaly leads to an increase of the critical dimension by two ($d = 28$ and $d = 12$ for N=0 and N=1 respectively), the no-ghost theorem requires one of these extra dimensions to be time-like, making a combination with a left N=2 string possible, and finally BRST invariance requires the $U(1)$ current to be a "null current". This means that these currents are of the form $\nu_\mu \partial X^\mu$ (plus $\nu_\mu \psi^\mu$ for N=1 theories) where ν is a light-like vector. The gauge symmetry then implies that physical excitations must have momenta in a plane orthogonal to the null vector ν_μ, and that all momenta that differ by ν_μ are identified. In this way one recovers 26 and 10-dimensional Lorentz invariance for (0,0) and (1,1) strings respectively. The (2,1) and (2,0) strings have two or three-dimensional Lorentz invariance, depending on the orientation of the vector ν^μ. We will only consider two dimensions here. For a discussion of the three dimensional case, given a modular invariant two-dimensional theory, we refer to Appendix A of [7].

The assumption that there are no twists in the null directions implies that, apart from the null-current constraint, the theories we consider are Lorentz-invariant in four dimensions. This constrains four bosons of the $c = 28$ N=0 sector and four bosons and four fermions of the $c = 18$ N=1 sector, leaving respectively a c=24 conformal field and a c=12 superconformal field theory undetermined. It is these theories we wish to classify.

One can see that the c=24 N=0 theory must be modular invariant by itself, and a classification of such theories is already available, as mentioned above. The c=12 superconformal field theory makes a contribution to the partition function that depends on the spin structure of the right-moving world-sheet fermions. It must thus be a theory with four characters with modular transformations dictated by those of the NSR fermions.

To classify the (2,1) strings we will make use of techniques similar to those used in [12] for the classification of ten-dimensional heterotic strings. The starting point is a bosonic formulation of all fermions with space-time indices and all bosonic ghosts, *i.e.*

all fields carrying non-trivial spin-structures. We emphasize that nothing is assumed about the other right-moving degrees of freedom, except that they should form a $c = 12$ superconformal field theory. We will in fact weaken this requirement to "conformal", and inspect superconformal invariance at the end.

For a ten-dimensional heterotic string the bosonization of ψ^μ and the β, γ ghosts yields a description in terms of a six-dimensional "covariant lattice" with a metric with signature $(+, +, +, +, +, -)$ [13] (see also [14-16]). The five positive metric fields correspond to the bosonized $SO(9, 1)$ NSR fermions while the last component corresponds to the bosonized superghosts (note that the lattice metric is not related to the space-time metric). If one ignores the metric, the lattice is just the weight lattice of D_6, but to indicate its metric we will call it $D_{5,1}$. The lattice structure is a direct consequence of the requirement that all 10 space-time fermions as well as the superghosts must have the same spin structures on any Riemann surface. The generalization to lower dimensions is straightforward, and in two dimensions one needs a two-dimensional lattice $D_{1,1}$ with signature $(+, -)$.

In the present case some changes are required, due to the null current and the extra ghosts. A useful guiding principle is the fact that such a formulation is also available for (0,1) strings, i.e. the usual heterotic strings, and should give the same answer. This alternative formulation is in (28,12) dimensions rather than (26,10), and includes null currents and extra ghosts. If one takes the equivalence between these formulations of (0,1) strings for granted the main result for (2,1) strings follows very easily. However we prefer not to take it for granted, and examine the new covariant lattice description more carefully. We will present this in such a way that the results are valid for a (compactified) (0,1) string in $2n$ flat space-time dimensions as well as a (2,1) string in 2 flat space-time dimensions (for $n = 1$).

The right-moving sector of such a string theory is built out of a $c = 12$ superconformal field theory, multiplets of bosons and fermions X^μ, ψ^μ, where μ is a space-time index of a space with signature $(2n, 2)$, plus ghosts. The ghost sector consists of reparametrization ghosts b and c, superghosts β and γ, plus fermionic ghosts b' and c' of the world-sheet $U(1)$ gauge symmetry and their bosonic superpartners β' and γ'. The following table summarizes the dimensions and total central charge of these fields.

fields	dimensions	central charge
(b, c)	$(2, -1)$	-26
(β, γ)	$(\frac{3}{2}, -\frac{1}{2})$	11
(b', c')	$(1, 0)$	-2
(β', γ')	$(\frac{1}{2}, \frac{1}{2})$	-1

In comparison with the right sector of the usual (0,1) string there are two extra

world-sheet fermions plus an extra bosonic ghost system (β', γ') that depend on the spin structure. World-sheet supersymmetry requires all these spin structures to be identical. Hence if we bosonize the fermions and the bosonic ghosts, the resulting two additional bosonic degrees of freedom can be taken into account by adding two dimensions to the covariant lattice, with opposite metric. So the right-moving fermionic sector is described by a covariant lattice $D_{n+1,2}$ plus the internal $c = 12$ conformal field theory.

In the one-loop partition functions the fermionic ghosts b, c, b', c' cancel the contribution of four bosons X^μ, reducing their contribution to the transverse one, which in two dimensions is trivial. The superghosts (β, γ) cancel the contributions of two fermions ψ^μ of opposite signature, exactly as in the (0,1) string, while the remaining bosonic ghosts cancel the contribution of the remaining two fermions ψ^μ. At arbitrary genus, the partition function has the form

$$\sum_\alpha (Det^\alpha_{1/2})^{2n} (Det^\alpha_{1/2})^{-1} (Det^\alpha_{3/2})^{-1} \mathcal{C}_\alpha \tag{1.1}$$

where α denotes the spin structure, and $\mathcal{C}$ the contribution of the unknown internal CFT. Note that the extra fermion and the extra ghost just cancel, since both are $(\frac{1}{2}, \frac{1}{2})$ determinants, albeit with opposite metrics. This cancellation is exact at arbitrary genus, unlike the cancellation of the spin-$\frac{3}{2}$ determinant, which occurs only for genus 1. The exact cancellation of the $\beta'\gamma'$ ghost contribution is essential for the exact equivalence of the two (0,1) formulations discussed above. Consequently the discussion of modular invariance is unchanged. Note also that the conjugacy classes on the $D_{n+1,2}$ lattice have the same norms (modulo even integers) as those of the $D_{n,1}$ lattice used in the standard formulation of two-dimensional (0,1) strings.

Consider now $n = 1$, either for (2,1) strings, or for (0,1) strings with a meromorphic CFT as their left-moving sector. Since the left sector is separately modular invariant, the combination (1.1) with $n = 1$ must be modular invariant as well. It is known (see e.g. [16]) that the ratios

$$Y^\alpha = \frac{Det^\alpha_{1/2}}{Det^\alpha_{3/2}} \tag{1.2}$$

transform in exactly the same way as

$$X^\alpha = (Det^\alpha_{1/2})^4 \sum_\beta (Det^\beta_{1/2})^8 \tag{1.3}$$

Hence the following combination must be modular invariant

$$\sum_\alpha (Det^\alpha_{1/2})^4 \mathcal{C}^\alpha \sum_\beta (Det^\beta_{1/2})^8 \ .$$

This combination can be interpreted as the partition function of a meromorphic $c = 24$ theory, since only determinants of ordinary fermions occur, and since $\mathcal{C}$ represents a unitary CFT.

The fermion determinants can in fact be interpreted as characters of the level-1 affine algebras D_4 and E_8. To read off the possibilities for the internal CFT one must look for meromorphic $c = 24$ CFT's that have $D_4 \times E_8$ at level 1 as a subalgebra. Due to the presence of the E_8 factor this problem reduces to looking for D_4 subalgebras of $c = 16$ meromorphic CFT's, and then the only possibilities are the familiar even self-dual lattices $E_8 \times E_8$ and D_{16}.

To obtain level 1, D_4 must be embedded in just one E_8 factor, and then there is just one possibility, namely the embedding $E_8 \supset D_4^{\text{ghost}} \times D_4^{\text{int}}$ defined by

$$(248) \to (28, 1) + (1, 28) + (8_v, 8_v) + (8_s, 8_s) + (8_c, 8_c) \tag{1.4}$$

On the other hand, for the embedding $D_4^{\text{ghost}} D_{12}^{\text{int}} \subset D_{16}$ there are three distinct possibilities, namely

$$(496) \to (28, 1) + (1, 276) + (8_v, 24) \tag{1.5}$$

$$(496) \to (28, 1) + (1, 276) + (8_s, 24) \tag{1.6}$$

$$(496) \to (28, 1) + (1, 276) + (8_c, 24) \tag{1.7}$$

These three embeddings are related by triality. One might think that the same possibility exists also in the first case, but there all triality rotated embeddings are in fact indistinguishable, because they can be undone by a compensating triality rotation in the second D_4 factor.

The spectrum is now easy to obtain using the rules formulated in [17]. These rules require some more discussion due to the extra components on the lattice. Again the (28,12) dimensional formulation of (0,1) strings can serve as a guiding principle. Considering again first (0,1) strings in arbitrary (even) dimensions $D = 2n$. The two formulations involve then covariant lattices $D_{n,1}$ and $D_{n+1,2}$ respectively.

Let us first review the argument for lattices $D_{n,1}$. In terms of lattices, the partition function map replacing Y_α by X_α (cf. (1.3), (1.2)) is equivalent to replacing $D_{n,1}$ or $D_{n+1,2}$ by $D_{n+3} \times E_8$, using a map on conjugacy classes rather then on individual vectors. Not all vectors on the $D_{n,1}$ lattice correspond to physical states, since the ghost charge can be changed by acting with the picture changing operator $e^{i\phi}T_F$. Here ϕ is the boson in terms of which β and γ are bosonized, and T_F the supercurrent. The simplest picture is the "canonical" one, in which the physical states are in one-to-one correspondence with the negative modes of the bosonic and fermionic oscillators acting on the ground state. To make sure that the positive modes of the bosonic ghosts β and γ annihilate the ground state one must assign a ghost charge q to it; otherwise the action of these modes would render the energy unbounded from below. As discussed in

[18], this charge is

$$q = \tfrac{1}{2} - \lambda \qquad \qquad \text{(NS)}$$
$$q = 1 - \lambda \qquad \qquad \text{(R)} .$$

$$(1.8)$$

for a general β, γ ghost system of conformal weights $\lambda, 1 - \lambda$. For the superghost system this yields $q = -1$ (NS) and $q = -1/2$ (R). To read off physical states one only considers vectors on $D_{n,1}$ whose last components are equal to one of these values. To read off the light cone states directly one may furthermore fix the second-to-last entry to 0 (NS) or $-\tfrac{1}{2}$ (R), so that one only considers vectors with last components equal to $(0, -1)$ or $(-\tfrac{1}{2}, -\tfrac{1}{2})$. All those vectors are precisely obtained by considering the complements of the conjugacy classes (v) and (s) of $D_4 \subset D_{n+3}$.

Similar considerations apply for the $D_{n+1,2}$ lattice, except that now there is a second picture changing operator associated with the β', γ' bosonic ghost system. In comparison with the lattice of the light cone rotation group D_{n-1} the covariant lattice $D_{n+1,2}$ has four extra components, corresponding respectively to

— (a) The extension from light cone rotation group $SO(2n-2)$ to the Lorentz group $SO(2n-1, 1)$.

— (b) The null current direction.

— (c) The bosonized β, γ ghosts.

— (d) The bosonized β', γ' ghosts.

Entries (a) and (c) are exactly as before.

The β', γ' ghosts can be treated completely analogously to the β, γ ghosts. Setting $\lambda = \tfrac{1}{2}$ in (1.8) we find now $q' = 0$ (NS) and $q' = \tfrac{1}{2}$ (R). This determines entry (d). Entry (b) is fixed by the null-current constraint on the vertex operator. For the NS and R massless states this fixes entry (b) to (0) and $(\tfrac{1}{2})$ respectively, *i.e.* the projection on the null current direction should respectively be a singlet or a spinor of definite chirality (the choice of chirality is irrelevant). We have verified that these ghost charge assignments and constraints are in agreement with BRST invariance of the vertex operator. They are also in agreement with [6].

Combining all this, we find thus that lightcone degrees of freedom can be read off by fixing the last four components to either $(0, 0, -1, 0)$ for NS or $(-\tfrac{1}{2}, +\tfrac{1}{2}, -\tfrac{1}{2}, +\tfrac{1}{2})$ for R. After mapping to D_{n+3} this corresponds precisely to the conjugacy classes (v) and (s) of D_4, exactly as in the other formulation in terms of $D_{n,1}$ covariant lattices.

Since we are considering two-dimensional target spaces, using light cone states is not as convenient as in higher dimensions. However, it follows from the foregoing discussion that one may also read off the covariant states directly by stripping off the conjugacy classes (v) and (s) of D_3 (rather than D_4). The massless spectra obtained from (1.4)–(1.7) are then respectively 8 (non-chiral) scalars + 8 chiral fermions + 8 anti-chiral fermions; 24 (non-chiral) scalars; 24 chiral fermions or 24 anti-fermions. These particles

can be assigned to representations of the internal group, which however is not dynamically realized. For (1.4) any permutation of the three non-zero conjugacy classes of $SO(8)$ is a possible assignment, whereas of course for (1.5)– (1.7) all states are in the vector representation of D_{12}. The correct counting of the fermionic states also follows from table 6 in [16], where the number of gravitini in (0,1) strings is displayed. In the case of (2,1) strings these fermions lose their space-time index μ and become spin-$\frac{1}{2}$ fermions.

So far we have not used the requirement of superconformal invariance, but only modular invariance and conformal invariance. If we had found new solutions, we should check that they are consistent with world sheet supersymmetry. However, these three cases are not new, and have all been discussed before in [4] and [5],[6],[7]. In these papers these solutions were obtained *assuming* a free fermionic description of the $c = 12$ conformal field theory. What we have shown is that this assumption is unnecessary, and that no other solutions exist, independent of a particular construction method.

The identification of the explicit target space lagrangian giving rise to the physical spectra we have found in three cases is more involved. The (2,0) world sheet theories were conjectured (in [5]) to lead to purely bosonic compatifications in target space. The system containing 24 physical seems ([7]) to represent a special (1,1) string theory. The other two spectra, the 8 bosons and eight fermions and the 24 chiral(or anti-chiral) fermions were associated with a type IIB string and a (2,1) string respectivly. The fact that a (2,1) system reproduces itself suggests some flow in the space of self-reproducing theories. Other work ([19]) suggests that actualy all (N, M) systems with $N, M < 5$ should be reproduced as target space descriptions of (2,1) worldsheet theories. In [7] it is claimed that (1,0) and (2,2) theories can be identified as well. The former is found using an orbifold twist in the null direction, a possibility which we have not considered, and the latter is obtained from a partition function that violates the spin-statistics relation. The $(2, 2)$ target space contains no physical states and is in fact a topological theory, so perhaps this is just what is needed, with the wrong-statistics fields interpreted as ghosts. However, in any case our method can not produce such a solution, since it is based on a map to a bosonic string partition function with positive signs, and since furthermore very little is known about modular invariants with non-definite signs. It also not at all clear how to the data on only the physical states will differentiate between (2,2),(3,3) or (4,4) topological systems. We wish to return to this problem in the future.

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